axsym engineering

TOWARDS A COMPREHENSIVE MACROECONOMIC SIMULATION CAPABILITY: ECONOMY DYNAMICS

William Maybury **January 2020**

This paper presents a new approach to macroeconomic forecasting to overcome limitations in the current use of semi-empirical functions correlated to historical datasets. The limitation of existing methods occurs when the input variables of the function being used for forecasting are beyond the range of values held within the correlated dataset. Thus, extrapolating the semi-empirical function introduces numerical uncertainty into the forecast results. This requires intervention by economics forecasters to process differences between model predictions and actual data from the recent past. Because the intervention process can be specific to the individual forecaster, two forecasters using the same model can end up with very different forecasts.

The new approach, called *Economy Dynamics*, avoids the limitation of extrapolating semi-empirical functions. The methodology has been developed independent of current economic theories. The concept has been based on observations of the decision-making and financial circumstances of businesses and households and defining the interdependency between consumers and suppliers. The models of the Economy Dynamics framework have been expressed using computational mathematics from engineering and data science. The framework architecture is based on recent advances in software architecture to automate complex analytical processes.

Economy Dynamics will be validated against metrics at a regional level such as economic output, poverty levels and inflation and against the environmental impact of consumer behaviour. The software architecture will enable any economic theory, that can be defined mathematically and fulfils model requirements of the Economy Dynamics framework, to be tested. Economy Dynamics will be used to evaluate the effect of substantial changes to fiscal and monetary policy on economic output, welfare and the environment. It will be used to numerically enhance data value ranges of historical datasets for improving the robustness of existing semi-empirical methods. It will also enable the study of the resilience of an economy to external events affecting essential supplies, such as regional political instability or resource depletion.

1 INTRODUCTION

Currently, macroeconomic analytical methodologies use predefined functions that are fitted to historical datasets to forecast economic output (see Blanchard, 2008). This approach can be described as top-down, as the macroeconomic assumptions of the model developer directly affects the conclusions of the model. Macroeconomic analytical methods have limitations in the degree of structural change to an economy that can be modelled (see for example OBR.uk, 2013). The limitation is caused by mathematical uncertainty when extrapolating functions beyond the range of values held within the correlated dataset. The use of a top-down methodology and the mathematical uncertainty of extrapolation means that it is not possible to validate economic theories independently of modelling assumptions. Therefore, where would someone start if they wished to advance macroeconomic theory?

To provide a constructive contribution to macroeconomic forecasting the new framework must augment the existing substantial body of work. The idealised objective is to identify a complementary approach that can validate the existing theories. The alternative to a top-down approach is to calculate economy performance from the ground up by modelling consumer and supplier behaviour and building up the economy simulation from the aggregation of the consumer-supplier interactions. Although a ground up approach may impose microeconomic model assumptions on the solution, this can be validated as long as it does not incur the type of mathematical uncertainty imposed by current methodologies. If so, the new approach will provide the opportunity to revise, fine tune or refute economic models until validation of the simulation is achieved.

To ensure the new approach can address the current forecasting limitation we need to evaluate the mathematical nature of these limitations. We can take as a general example the large scale macro-economic model, such as the method defined by OBR.uk (2013). Once the limitations have been determined we can draw upon analytical methodologies from other academic disciplines to achieve the required innovation in the macroeconomic forecasting methodology.

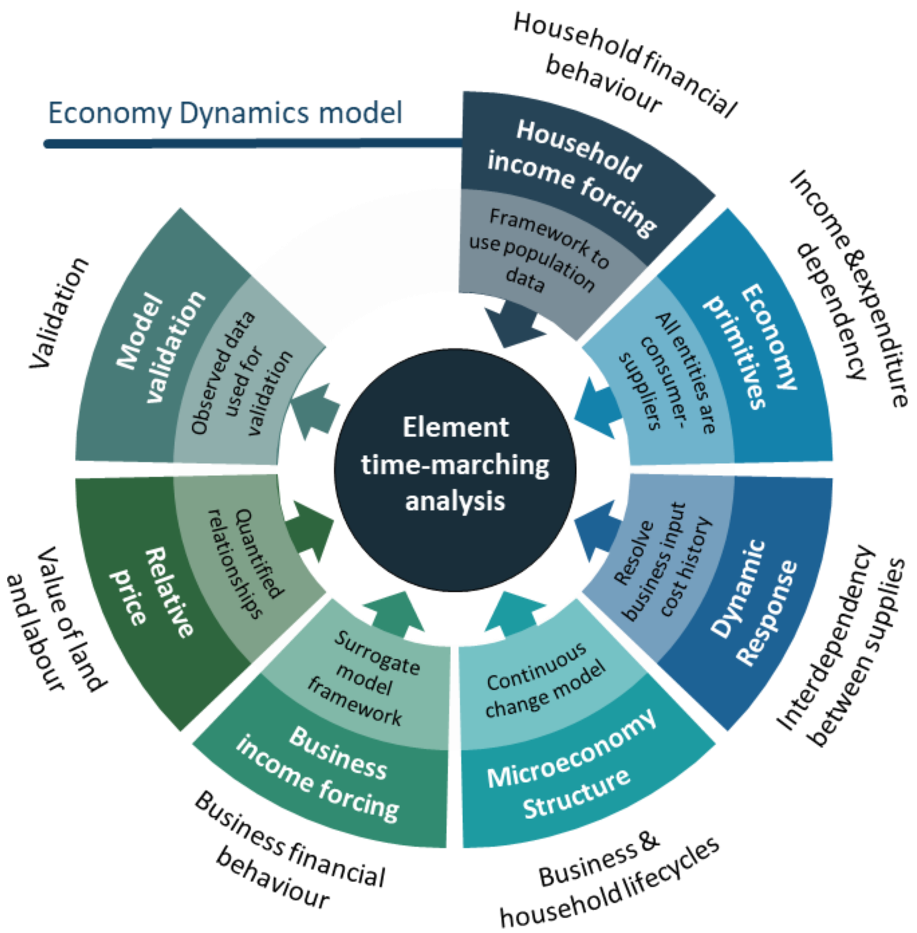

Figure 1. Economy Dynamics infographic

This paper presents an overview of a new methodology for macroeconomic analysis and forecasting. Economy Dynamics is a computational framework that will enable economic theories to be validated against observed features of an economy. The new approach will be presented in nine papers, each explaining the microeconomic and macroeconomic functionality needed to build the framework. The different components of Economy Dynamics are summarised in Figure 1 and the nine papers are listed in Appendix A.

A ground up approach must be defined by someone who has detailed understanding of the supplier and consumer perspectives of the economy, which does not necessarily require an academic economics background. The consumer and supplier perspective of an economy has been established from the author's experience as the founder of a business-to-business engineering service provider. This has enabled the close observation of operational processes and financial circumstances of partner businesses, competitors and clients who span both the public and private sectors, and range in size from sole traders to multi-national enterprises. Similarly, the author has observed a broad range of household financial circumstances. The perspective of businesses and households is not exhaustive but will be sufficient to provisionally define the general modelling requirements.

The computational mathematics used to define the microeconomic models in the new methodology are based on engineering and data science analytical techniques that have been used and developed by the author for over 20 years. The concept for the framework architecture, enabling versatile and adaptable simulation capability, is based on recent advances in software architecture to automate complex analytical processes (co-funded by Innovate UK). The automation of analytical processes has been demonstrated for transport network management (Moffatt, 2020) and local energy system design (Moffatt, 2019) and aerospace system design. The proposed Economy Dynamics methodologies presented in each paper in Appendix A are not claimed to be complete solutions, but are a starting point for further development or for substitution with existing theories. The analytical framework will be developed to enable adaptable simulation setup so that a forecaster can substitute any microeconomic model with any other equivalent methodologies. The aim is to provide a software architecture that will enable continual development of the analytical theories until the simulations can be validated against all required observed features of an economy.

2 CURRENT ANALYTICAL APPROACH

Macroeconomics forecast simulations currently use a semi-empirical analytical approach. This approach creates a relationship between a set of input variables and a given output variable through statistical correlation using real world data. For example, large scale macroeconomic models [e.g. OBR.uk, 2013] determine household consumption using predefined functions to establish the relationship between the input values of household income and wealth and the output value for their consumption behaviour. The type of function is specified by the macroeconomic model developer, which is logarithmic for household consumption in the OBR's model [OBR.uk, 2013]. The values of the function parameters are tuned by data analysts who fit the predefined function to historical data. The role of the forecaster is to decide how to process unaccounted residuals, which are the difference between the function's forecast and real data from the recent past. The occurrence of residuals is illustrated in Figure 2. The example predefined function overlays the historical data (from ONS.gov.uk, 2019) used to fit the function (on the left). However, the function forecast evaluated against data from the recent past produces unaccounted residuals (on the right).

Household consumption is one of numerous prescribed behavioural functions within the large-scale macroeconomic model, all of which have been fitted to historical data. This necessitates considerable human judgement in setting up the analysis to account for differences between the function forecast and data from the recent past. The need for human judgement hinders analysis repeatability as two forecasters using the same model can end up with very different forecasts dependent on their handling of the unaccounted residuals. This applies to all the different semi-empirical approaches for macroeconomic analysis discussed by Blanchard (2008) in his review on the 'State of Macro'. The potential factors contributing to the unaccounted residuals are discussed in each of the subheadings below.

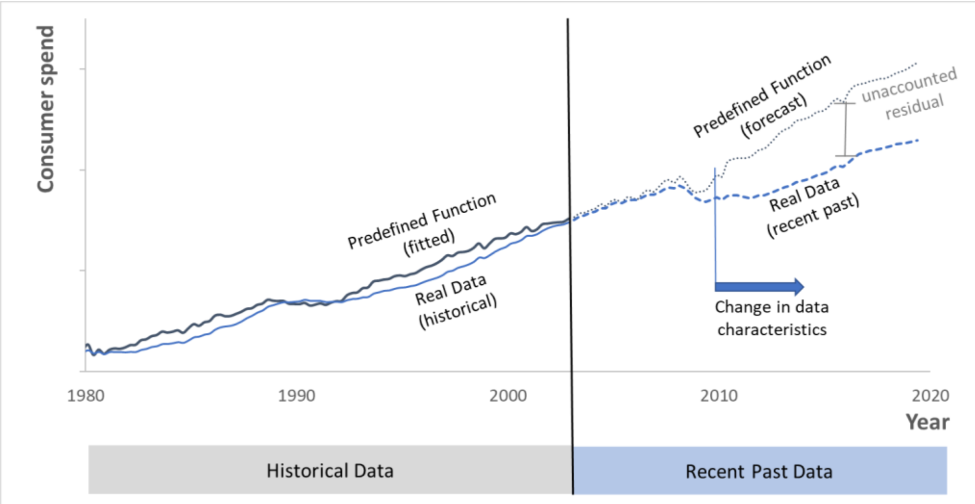

Figure 2. Illustration of unaccounted residuals in consumer spending

2.1.1 SAMPLE RANGE ERROR

A structural change in an economy, that is not reflected in the historical dataset, may result in extrapolating a trend that is poorly represented. For example, consider that a function, $f(x) = y$, is fitted to historical data from time period v. If at time w there is a structural change in the economy that requires a different range of values for x, then the function prediction will be offset by error ϵ. Thus, $f(x_t) = y_t$ when $v < t \leq w$, but $f(x_t) = y_t + \epsilon$ when $t > w$. Furthermore, the magnitude of ϵ will increase the further x is from the historical data range, therefore $\epsilon = f(x_t - x_v)$.

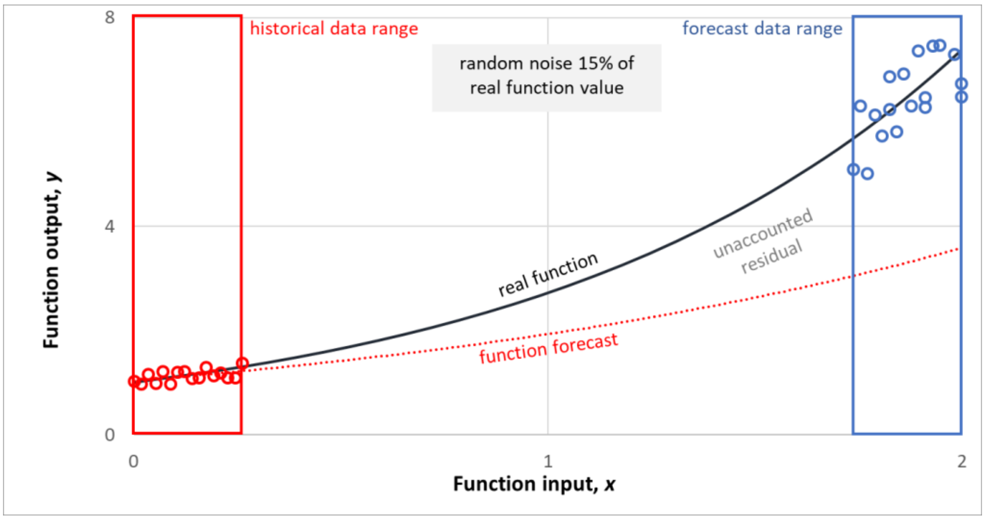

Figure 3. Illustration of sample range error

Figure 3 illustrates the sample range error by considering a relationship between x and y as an exponential curve (black solid line). The historical data range is $0 \leq x < 0.3$ (red open circle) and is used to regression fit an exponential function for forecasting y (red dotted line). After a structural change in the economy the new data range of interest is $2.7 \leq x < 3.0$. The data range error is the difference between the forecast and the real data y, which is an unaccounted residual. In addition, the magnitude of the unaccounted residual increases (as a relative proportion of the real function) the further the forecast data range is from the historical data range.

2.1.2 UNDERFITTING

Another sample range error is the risk of using an unsuitable function to fit historical data. The selected fitting function may adequately represent the relationship between x and y within the range of values for x in the historical data, but beyond this range the function rapidly diverges from the actual trend. Figure 4 illustrates this effect by considering a relationship between x and y as a sine curve. If the historical data range is $0 \leq x < 0.3$, shown in red, then the curve fitting function appears to be linear. If the historical data range is $0 \leq x < 2.4$, shown in blue, then the curve fitting function appears to be quadratic. The red dotted linear forecast and the blue dashed quadratic forecast deviate from the real function, shown in black, as the value of x increases beyond the historical data range used for curve fitting.

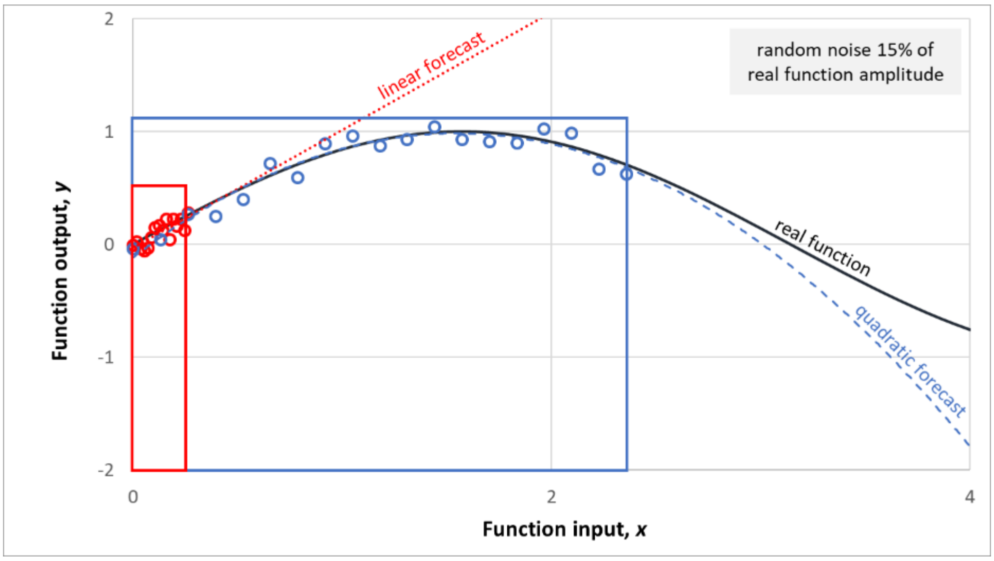

Figure 4. Illustration of underfitting

2.1.3 DATA AGGREGATION ERRORS

The output of a function using aggregated input data may not be the same as the sum of the outputs using unaggregated input data. For example, consider that economic variable y is a function of x. If the input variable value is dependent on geographic region, x_i (i from 1 to n), then a function that cannot use aggregated regional input values can be expressed as $y = \sum_{i=1}^{n} f(x_i) \neq f(\sum_{i=1}^{n} x_i)$. If this type of function is fitted to aggregated historical data there will be unaccounted residuals within the fitting process. Figure 5 illustrates these aggregation errors using an example with 25 distinct regions. At timestep 0 the value of x_i in each region is a random number between 0 and 2. Then for 20 sequential timesteps the value of x_i in each region is calculated by factoring the previous value of x_i

by a random number between 0.97 and 1.06. The output of each region at each timestep is $y_i = e^{x_i}$. The output of the economy at each timestep is $y = \sum_{i=1}^{n} y_i$ and is shown as black circles. The forecast function, in the form $y = ae^{bx}$ where a and b are determined by regression fitting using aggregated input values, is shown as red circles. The aggregation error, which are unaccounted residuals in the fitting process, are shown as grey error bars between the black and red data series.

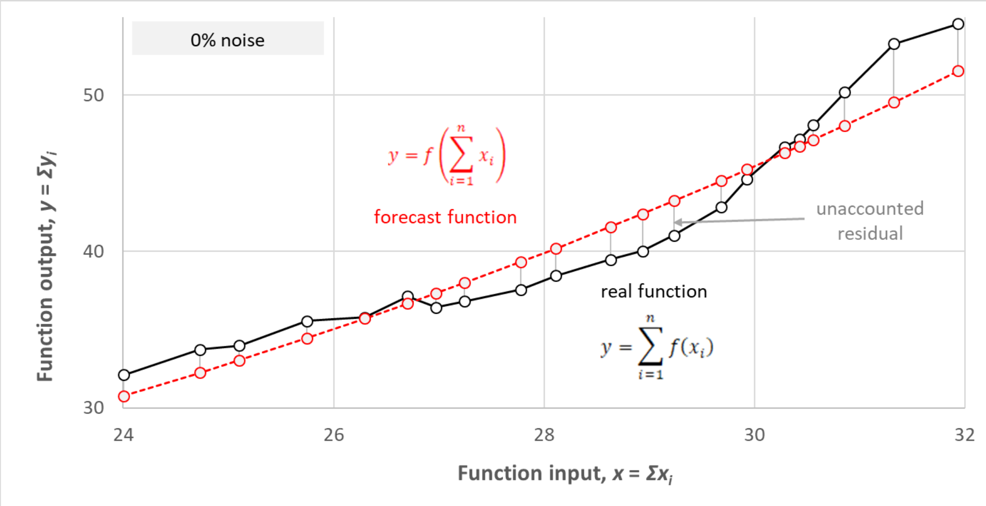

Figure 5. Illustration of aggregation errors

2.1.4 OMITTED VARIABLE BIAS

When statistically important variables are omitted from the function fitting process, unexplained variance in the data will be attributed to any variables that have a correlated relationship with missing variables. For example, if y is defined by parameters a, b and c, and independent variables x and w with error ϵ, then $y = a + bx + cw + \epsilon$. If w is defined by parameters d and g and the independent variable x with error σ, then $w = d + gx + \sigma$. In the event that w is omitted from a regression between x and y the apparent relationship between x and y will be equivalent to substituting w with its function of x, thus $y = a + bx + c(d + gx + \sigma) + \epsilon$. This will lead to estimating the relationship between x and y as $(b + cg)$ and not the correct value of b. In addition, the intercept parameter will be $(a + cd)$ and not the correct value of a and the error will be $(c\sigma + \epsilon)$ and not ϵ. The impact of this mistake may be overlooked in conditions where y is dominated by bx, such that $y \to bx$, whereas this is problematic for conditions where y is dominated by cw, such that $y \to (b + cg)x$, intercept $a \to (a + cd)$ and error $\epsilon \to (c\sigma + \epsilon)$.

2.1.5 SPURIOUS VARIABLE CORRELATION

A risk with multi-variate regression fitted models is that variables could provide a reverse correlation. For mediating variables where the direct causal relationship is actually $f(x) = w; f(w) = y$, the relationship may erroneously provide correlation of $f(w) = x$ and $f(w) = y$. If a spurious correlation occurs then the variance in the data explained by the mediating variable w increases while the data variance explained by the initial independent variable x decreases. This is a particular risk with macroeconomic modelling where the cause and effect relationships are highly convoluted, such as the link between economic output and employment, discussed in Paper 02 (see Appendix A).

2.2 *ALTERNATIVE ANALYTICAL METHODOLOGIES*

Addressing the uncertainty imposed on a forecast due to limitations within the historical dataset requires restructuring the methodology to achieve a different approach to using predefined functions. The predefined function must only be used for interpolation, ensuring that the forecast is within the input variable range of the historical data. This avoids numerical uncertainty caused by extrapolation. This would mitigate for both sample range errors and underfitting errors.

To minimise aggregation errors the model should be discretised, such as dividing an economy into distinct geographical regions based on rates of taxation, the costs of supplies, the cost of land and the cost of labour. The aim is to identify regions where the financial characteristics can be considered homogenous. The effect of discretisation will require evaluation, and this is performed for different levels and patterns of division using the same forecast model inputs. The objective is to determine if there is convergence of the solution output variable value with increasing discretisation.

Omitted variable bias and spurious variable correlations do not directly affect a ground up forecast. Naturally, any predetermined functions have the potential to suffer from variable omission and spurious variable inclusion. For a ground up approach, using predefined functions, the issue is at the economy element level. This provides a large sample of diverse economic conditions to test hypotheses against. Therefore, the aim is to define functions that maximise the explained variation in population data across all economy elements. This process becomes part of the continual development of the forecasting methodology, enabling revision and substitution of functions to provide the most accurate numerical solution.

2.2.1 ANALOGOUS USE OF PREDEFINED FUNCTIONS

Engineering has examples of analytical methods that use prescribed functions that avoid the limitation of numerical uncertainty by their implementation within a ground up discretised approach. One of the earliest examples of this is by Drzewiecki (1892) using the method now known as the blade element momentum theory *BEM* for fluid dynamics. The analytical principles of BEM have not changed significantly. In each element predefined functions calculate the underlying physical properties, such as the forces acting on a propeller blade element. Then the physical properties of all elements are integrated using system specific applied physics equations which produce updated input variable values for the predefined functions, such as the velocities in the wake. The computation is iteratively resolved between the predefined functions and the system specific applied physics equations until the solution reaches convergence.

The ground up approach avoids extrapolation as the range of input values for all predefined functions are always within know limits. Thus, the system performance is constrained to physically achievable conditions imposed by the ground up approach. In addition, discretisation of the system into elements is essential to achieve good solution fidelity. For example, BEM uses non-linear predefined functions and the function input variable values change significantly between the elements.

By way of analogy to macroeconomics, a downturn in manufacturing will have a different impact on different regions within an economy. This will be determined by the regional dependency on the industry for employment. A region whose main employment is within the downturned manufacturing sector could suffer high levels of job losses, which may have knock-on welfare issues. This will not be the case for other regions within the economy. Another analogy would be to consider the financial decision-making of households relative to a change in monetary policy. A significant change in the central bank base rate will have a different effect on households with mortgages to those without mortgages; will have different effects on households with fixed rate mortgages to those with variable rate mortgages; will have different effects on households who rent to those whose accommodation is provided though the benefit system; and will have different effects on households who own their property outright to those who do not.

We would not expect the economic forecasts of the USA, China, Japan, Germany and the United Kingdom to be equivalent. However, within each economy there is large regional diversity in economic activity. For example, economies have regions dependent on one sector for income, such as agriculture, tourism or manufacturing. As the behaviour of economies is not equivalent then the regional behaviour must also be treated differently. Therefore, macroeconomic models must be discretised both regionally and across consumer-supplier entities to provide a higher fidelity approach to forecasting.

2.2.2 FULL-PHYSICS APPROACH

Recently, generalised engineering analytical methods using a discretised full-physics analytical approach have emerged, such as Turner *et al.* (1956)[1]. These methods simulate engineered systems from the first principles of physics and not using the application specific applied physics equations. A full-physics approach expresses the underlying first principles of physics as the model's primitive equations. The computational geometry is built to be spatially representative of the simulation domain and is discretised to form numerous small discrete volumes or elements. The primitive equations are solved within each element to calculate physical characteristics taking account of the effects of the neighbouring elements.

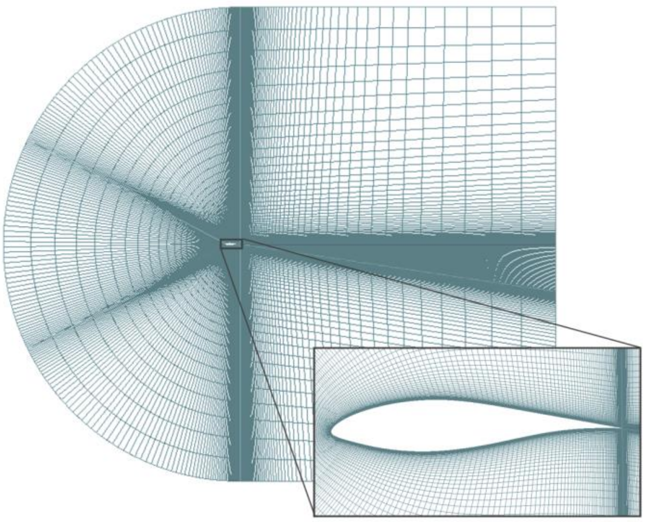

Figure 6. Finite element discretisation of the flow domain around the surface of a wind turbine blade section

[1] R. W. Clough, a co-author, is attributed to subsequently coining the term *finite element method* (FEM).

A full-physics simulation requires judgement in determining the discretisation of the simulation domain and judgement to define the conditions at critical interfaces within the system. Experienced analysts using full-physics simulation methods, such as structural engineers using finite element methods, will produce similar results. An example of discretisation to analyse the aerodynamic properties of an aerofoil is shown in Figure 6, which is used to determine lift and drag characteristics required for blade design.

For the new approach to macroeconomic forecasting we need to determine the financial circumstance of all consumers at each timestep in a simulation. To avoid extrapolation of prescribed functions, this requires the interdependency between income and expenditure of all individual entities within an economy to calculated. There are two approaches to achieve this:

- define high level solution integration functions. These are similar to the system specific physics functions used in the BEM approach. The methodology must iteratively integrate the macroeconomic solution and then update the input variable values of the prescribed behavioural functions in each element until the macroeconomic forecast has converged.

- identify the microeconomic primitive equation. This is similar to the full-physics primitive equations that are to be solved for all discretised elements (i.e. the economy elements for all consumer subgroups). An iterative process will be required to resolve the supply price (as discussed in Section 3.1.5). Calculating the financial circumstances of all consumer entities will be a time stepping solution without iterative convergence processes between timesteps.

It is possible that the former approach may be achievable given detailed knowledge of macroeconomic systems. However, there is a risk of imposing a top-down limitation on the solution if existing methodologies are implemented for this purpose. Therefore, based on the author's knowledge of the economy from a business and household perspective, this paper will identify the form of the primitive equation.

2.2.3 Full-physics methods that use approximation functions

For simulation expedience in some finite element methods, approximation functions can be used to avoid extremely fine discretisation that would lead to excessively high computing costs. The approximation functions are equivalent to semi-empirical predefined functions. In computational fluid dynamics *CFD* (including those used meteorological analysis) the fluid is represented by the primitive equations for the conservation of mass, the conservation of momentum and the conservation of energy. In CFD analysis approximation functions are used to model fluid flow behaviour in the boundary layer. The approximation functions are specific to particular boundary layer characteristics and their selection is made by the analyst. However, as long as the approximation function is appropriate for the given conditions and critical interfaces within the system are set up following best practice, then the simulation will generally achieve a solution representative of reality.

The combined full-physics and approximation function approach is a close analogy to the Economy Dynamics framework. The macroeconomic primitive equations determine the financial circumstances of all consumers (see Section 3.1.4) and the supply and demand functions resolve supply prices (see Section 3.1.5), similar in approach to a full-physics methodology. However, the consumer-supplier behavioural models will be similar to approximation functions (see Section 3.1.2), as it is not computationally practical to simulate the behaviour of every individual household and business. In addition, as consumer behaviour will need to be modelled using a probabilistic approach then aggregating consumers into subgroups of similar behaviour is unlikely to significantly reduce solution fidelity. It may also provide a useful structure to the study of population behaviour.

2.3 *LIMITATION OF ANALYTICAL METHODS*

Semi-empirical methods are restricted in their application to the range of input variable values captured within the training dataset. Discretised full-physics methods are constrained by the physics represented in the primitive equation. When using approximation functions with discretised full-physics methods these are restricted in their application to ensure the functions are only interpolated. The process of model discretisation requires studies to establish best practice for the level and pattern of discretisation. The discretisation process must then follow the best practice guidelines. Given these constraints are met, then by using either semi-empirical or full-physics methods, a system can be studied under conditions that could not be measured or demonstrated using tests of a real system or of a physical model.

It may seem counter-intuitive that we can extrapolate the conditions of a system beyond that of the historical or measured datasets, whilst ensuring that the prescribed functions are only used for interpolation. The premise of Economy Dynamics is that the financial behaviour is determined by the specific financial circumstances of the individual entities and not the state of the economy. The financial behaviour of households and businesses will be modelled from population data. Some entities will experience negative financial circumstances regardless of the structure or growth of an economy. Therefore, within historical population data there will be a sufficient range of financial circumstances to allow for the simulation of household and business behaviour, independent of the overriding characteristics of the economy, without extrapolation of historical data.

3 APPLIED MATHEMATICAL MACROECONOMIC MODEL

This section describes an applied mathematical methodology for macroeconomic analysis. This is a ground up approach to forecasting that simulates the aggregated financial behaviour of consumers. As consumers are also suppliers of goods, services, assets or labour their financial situation is the difference between their income and expenditure. The consumer behaviour is determined by their financial situation and their behavioural characteristics.

Consumers will be divided into subgroups that have similar financial features and behaviour characteristics. The financial situation of consumers is determined by the dynamic interaction between all consumer subgroups as consumers and suppliers within an economy. The dynamic interaction is affected by fiscal policy, monetary policy and the availability of essential supplies. The aggregate of consumer behaviour will determine the economic output, and the dynamics of this behaviour will determine household welfare and environmental impact.

The following subsections define the Economy Dynamics framework, comprising the components of the methodology and the analytical process to generate macroeconomic forecasts.

3.1 *FRAMEWORK STRUCTURE*

The framework structure comprises computational objects and dataset indexing, functions to interlink the income and expenditure of all consumers, behavioural models and consumer lifecycle models.

Many of the proposed components have analogous existing economic theories. The framework structure, embedded within an advanced software architecture, will enable validation of the overall framework using different microeconomic models and macroeconomic equations. However, to present the framework in a form that achieves the applied mathematical aim, new models and equations will be discussed and defined based on the author's engineering and data science experience.

3.1.1 Supplies

The goods and services computational object i_b represents the basket of all supplies traded within the economy, including consumable supplies, depreciable assets, financial products, land, property, equipment, private business stakeholding and intellectual property, as well as all goods, services and assets traded in the undeclared economy without paying tax. As a computational object of all supplies it defines the supply and demand characteristics, the resolved supply cost price and the resolved environmental parameters such as the supply carbon footprint. A detailed description of the supply object will be provided in Papers 03 (see Appendix A).

3.1.2 Consumers and suppliers

Consumers must have a budget that can be used to purchase supplies and assets. For sustainable consumerism all consumers must generate an income that forms the main constituent of their expenditure budget. To enable the primitive equation (Section 3.1.4) to be in an identical form for all consumer types, the definition of the consumer types is critical. The four consumer types are differentiated by their income and expenditure transactions and are called households, businesses, public purse (or tax) and charity (or charitable funds). To maximise the explained variation in population data, consumer type is divided into subgroups that have similar financial behaviour (and similar income generating activities). The definition of the four consumer types and their respective subgroups is summarised below and a full description of households is provided in Paper 01 (see Appendix A). A full description of businesses, the public purse and charity will be provided in Paper 05 (see Appendix A).

3.1.2.1 Households

Households are the financial unit of interdependent human legal entities that provide the economy with the resource of labour. Households cannot sell goods they have produced nor can they sell services they provide. If the household sells goods or services they have created, then the income generated is treated as business turnover under the business category of self-employed. Households can sell assets, including land, property and durable supplies that have depreciable residual value. The main income for the majority of households is payment for labour.

The household population of an economy element is divided in subgroups i_{e+} that have similar standards of living and are further subdivided by likely financial characteristics and behaviour, based on composition of the household, the skill category of the main income provider, the stage of family and working life based on the age of the main income provider, their minority grouping and their parental living standard. Household subgroups are defined in detail in Paper 01 (Appendix A).

3.1.2.2 Businesses

Businesses are non-human legal entities that produce goods or services in exchange for money. Businesses include all supply producing organisations in both the public and private sectors that can be either for-profit or not-for-profit organisations. In contrast to current economic models, the role of banks and government organisations, in addition to charities, households selling new supplies and households with employer responsibility, are grouped within the business category.

The business population of an economy element is divided in subgroups i_{B+} that have a similar business type based on output supplies and are further subdivided by the enterprise class, the stage of business operations based on the business age, brand differentiation subgroups based on supply quality and ethics of business operations and multi-element indexing for large national and multi-national organisations. Business subgroups will be defined in detail in Paper 05 (Appendix A).

3.1.2.3 Public Purse

The public purse, also referred to as the tax consumer, reflects fiscal policy. Fiscal policy must determine the collection points of tax, represented within the macroeconomic primitive, which can be differentially distributed between legal entities. Fiscal policy must also set the distribution of tax spending in benefits to households, businesses and charity in domestic and foreign economies and to supplement tax spending by foreign governments and pay fees for membership of international coalitions. Subdivision of the tax consumer type i_G is identified by the fiscal policy responsibility of the different government organisations at local and national (and federal) levels for collection and distribution of public purse funds. Public purse subgroups will be defined in detail in Paper 05 (Appendix A)

3.1.2.4 Charity

The charity consumer type reflects charitable fund generation and the distribution of charitable funds as benefits to households, businesses and charity in domestic and foreign economies and supplementing tax spending by foreign governments. Subdivision of the charity consumer type is identified by the purpose of the charity i_C. Charity subgroups will be defined in detail in Paper 05 (Appendix A)

3.1.3 ECONOMY ELEMENTS

To reduce or eliminate aggregation errors the Economy Dynamics framework requires the economy to be separated into distinct geographical regions based on rates of taxation, the costs of supplies, the cost of land and the cost of labour. These regions, known as economy elements, are identified by index i_α for households, i_β for businesses, i_τ for the public purse and i_ϕ for charity funds. Therefore, the economy is discretised into economy elements where the financial characteristics can be treated as homogenous across the economy element (i.e. the geographical region) for a given consumer type. Economy elements will have a lower size limit based on the consumer type population size required to implement the probability distribution of financial behaviour. The upper size limit will be determined by the best practice (to be studied) to avoid aggregation errors. A detailed discussion of using economy elements to resolve the purchase price based on supply and demand of goods and services, land and labour will be provided in Papers 03, 06 and 07 (see Appendix A).

3.1.4 PRIMITIVE EQUATIONS

Businesses have a pivotal role within the continuous dynamic behaviour of an economy as a consumer and producer. As all entities within an economy are consumers it is possible to represent all financial interactions by a single diagram centred on the consumer relationship to suppliers of goods, services, labour and assets (see Figure 7). This enables a new approach to model the dynamic interactions of consumers and suppliers within an economy. Termed the *macroeconomic primitive* it defines the interdependency between budget and expenditure of all consumers. The macroeconomic primitive enables the financial circumstance of all consumers to be assessed at each timestep in a simulation. The primitive will need to be adjusted appropriately to reflect the fiscal policy of the simulated economy. The primitive equation enables the macroeconomic simulation to follow an applied mathematics approach. A detailed discussion of the macroeconomic primitive is provided in Paper 02 listed in Appendix A.

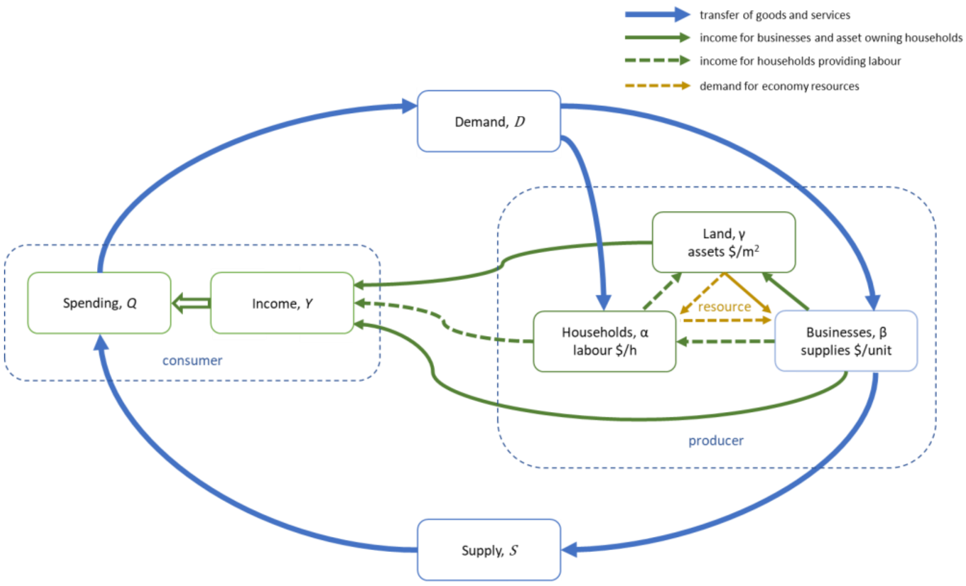

Figure 7. Continuous dynamic cycle has asymmetry of household dependency on business.

3.1.5 PRICING

The supply and demand relationship for supplies must be resolved between the consumer demand and the supplier output within the different respective economy elements for the four consumer (and supplier) types. The cost price of goods is dependent on the economy element in which the product is manufactured, or service is provided, due to the cost of labour and cost of land. The cost price is also dependent on the date at which, and economy element from which, input supplies and assets were purchased. Therefore, the supply and demand solution must account for the production costs and input supply time history and location costs. The dependency on time and interdependency on location to determine a supply cost price is represented in Figure 8. The methodology to resolve the economy dynamic response to net local supply and demand will be defined in Paper 03 (Appendix A). Similar to resolving the sales price of supplies, the value of land and the value of labour will require equivalent methodologies that will be discussed in Papers 06 and 07 (Appendix A).

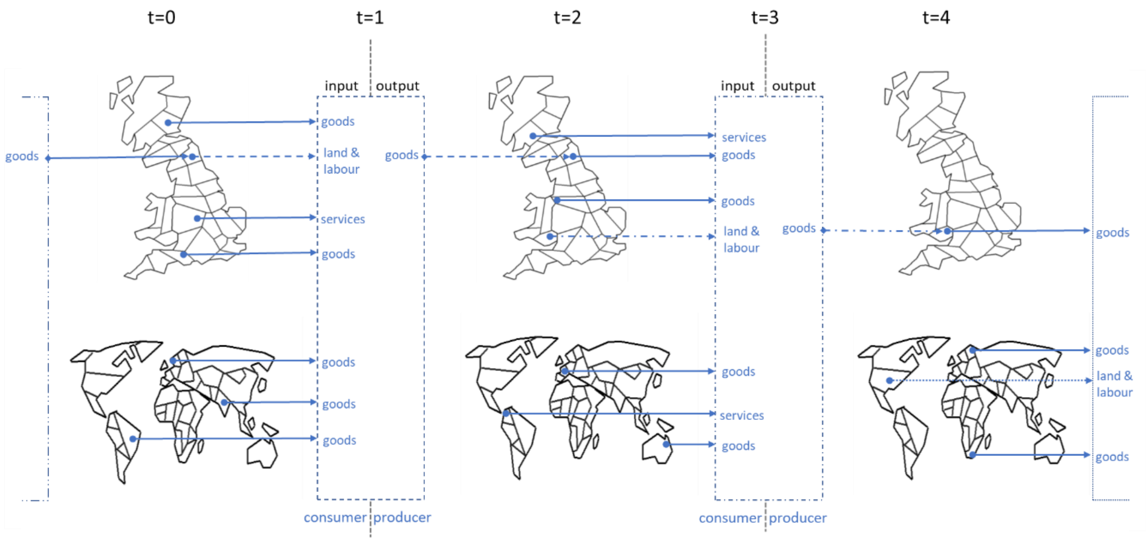

Figure 8. Resolving supply cost price for regional and temporal supply inputs

As time history and regional factors are included within this approach, the effects of resource depletion, the discovery of new and alternative resources, the influence of political unrest, the effects of crop failure and other such events effecting input supply availability can be incorporated into the simulation of the input supply cost price. This will quantify commercial advantages of the economy element selected by businesses for supply production and business operations.

3.1.6 Financial Behaviour

Surrogate model is a general name for the group of analytical methods that include machine learning algorithms (such as artificial neural networks), regression fit hypersurfaces (such as radial basis functions) and regression fit statistical models (such as a general linear models). All surrogate models have parameter values populated by training or fitting the model to an existing dataset. Once fitted the model can 'predict' the response of the system that produced the training dataset for a given set of input variable values. Ideally, the input variable values must be within the range of the historical dataset, thus using the surrogate model for interpolation and not extrapolation.

It is proposed that surrogate models trained against population data are used to predict the financial behaviour of households and businesses. The distinct subgroups within households and businesses must be established to maximise the fit of the surrogate models to the historical datasets of population financial behaviour. The subgroup categories listed in Section 3.1.2 may need refining, or alternative approaches can be used, to achieve the most complete explanation of the variation in population financial behaviour when fitting the surrogate model. The household and business financial circumstances are determined by their income relative to their running cost (discussed in Papers 01 & 05). The financial circumstance provides the input values to enable the surrogate model to predict the household or business financial behaviour. Figure 9 shows an analytical process for resolving financial behaviour within the household subgroups for each economy element, which is required before resolving the supply and demand function. Household financial behaviour is discussed in Paper 01 and business financial behaviour will be discussed in Paper 05 (see Appendix A). The behaviour of the public purse and of the charity funds will also be discussed in Paper 05.

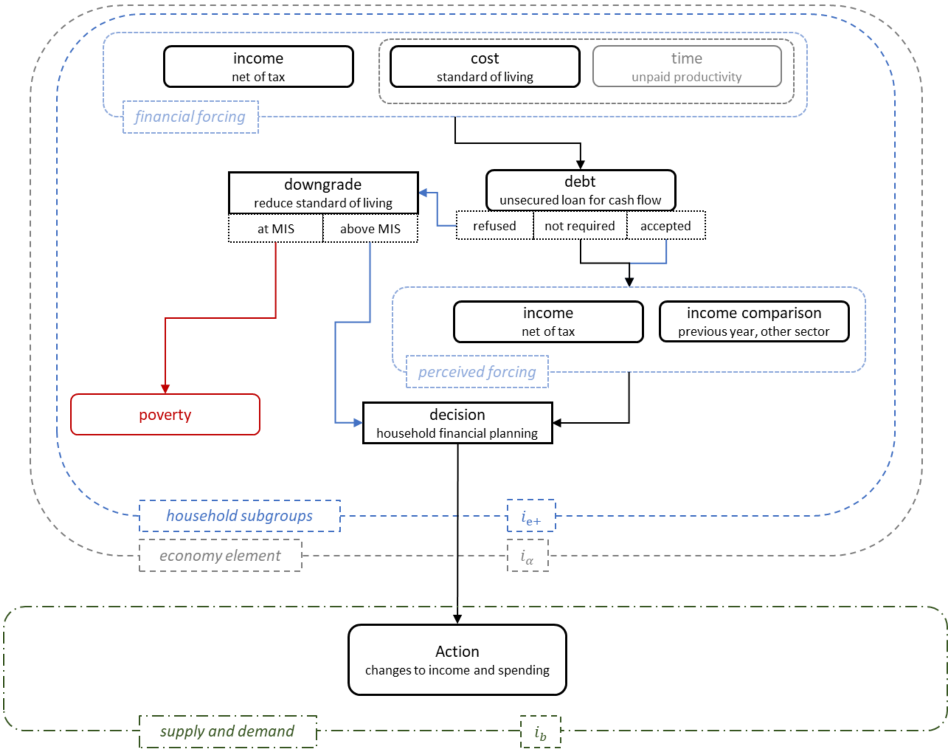

Figure 9. Financial forcing impact on decision-making for households

Note that fiscal policy changes affecting the public purse must be implemented through the primitive equation (Section 3.1.4) and the change in supply and demand behaviour (such as nationalising private sector companies or privatising public sector companies) must be implemented through a behaviour model of the public purse consumer type.

Speculation behaviour of a consumer (and supplier) can be implemented within this framework if required. The simulation would test the value of future prices by running the Economy Dynamics simulation forwards in time without speculation functions. The model could then apply a probability function against the accuracy of the speculation behaviour. The simulation would be rerun, applying the probability function to the relevant speculation behaviour of consumers and suppliers, so the effect of speculation on the economic outcome can be evaluated.

3.1.7 MICROECONOMIC LIFE CYCLES

Life stages of households and businesses impose change into the economy's continuous dynamic cycle. Similar to household and business financial behaviour, the life cycle events of the consumers can be modelled using surrogate models trained against population data. The purpose of the distinct subgroups within households and businesses must be to maximise the fit of the model to the historical datasets of population life cycle events. The life cycle modelling will be discussed in Paper 04 (see Appendix A). A summary of the life cycle events is provided below for households and businesses.

3.1.7.1 Household life cycles

Some household life cycle events will be captured within the financial behaviour models, such as people within a household looking for a new job. This type of event could also be captured by the life cycle surrogate models. Therefore, for some life cycle events there are two approaches to implementing the event. One approach is to allow the simulation to calculate and impose the event on a household (because their income is less than their running cost) and the other approach is to use the life cycle surrogate model to impose the event, regardless of the financial behaviour model. However, where possible to minimise the risk of surrogate model extrapolation (imposing a behaviour that has not be calculated by the model), it is preferable to allow these events to be implemented through the financial behaviour model. The main household life cycle events include:

Births:	new dependents added to the system and allocated to household subgroups;
Transitions:	educational stages, employment and career stages, such as dependents join workforce;
Migration:	moving the household to a different economy element or commuting to work in another economy element;
Unions:	two households merge through marriage, civil partnership or cohabitation;
Amalgamations:	adult relatives and non-relatives join households, such as adult children returning to the parental home unable to find work or parents living at the home of an offspring after retirement;
Separations:	single households become pairs through divorce or separation;
Dispersions:	adult relatives and non-relatives leave household to form a new household;
Deaths:	dependents and workforce leave system, and redistribution of assets to beneficiaries and tax to the public purse.

3.1.7.2 Business life cycles

Some business life cycle events, like the household events, will be captured by the financial behaviour model, such as businesses going bankrupt. This type of event could also be captured by the life cycle surrogate models. Therefore, for some life cycle events there are two approaches to implementing the event. However, where possible to minimise the risk of surrogate model extrapolation, it is preferable to allow these events to be implemented through the financial behaviour model. The main business life cycle events include:

Start-ups:	new businesses joining a business group;
Transitions:	business grow into a new enterprise group;
Migration:	moving the business operations to a different economy element or business head office to a different economy;
Consolidation:	multiple competing businesses merge;
Merge:	multiple complementary businesses merge;
Separations:	concentrate on core supply selling off non-core activities;
Spin-off:	separate non-core activities into start-up;
Bankruptcy:	business failure, job losses and redistribution of assets to creditors.

3.1.8 WELFARE AND ENVIRONMENTAL METRICS

The Joseph Rowntree Foundation has proposed a minimum income standard *MIS*, which is the lowest income needed to maintain basic human dignity (Bradshaw et al., 2008 and Hill et al., 2016). This benchmark income is identified by determining the minimum basket of goods and services required to live and participate within a community, specified by people from the community who are guided in their choices by welfare experts. As household financial behaviour will be affected by the difference between their living costs and their income, and the MIS is the lowest living costs in an economic element, then welfare is quantified by the Economy Dynamics framework. Furthermore, welfare will be quantified using the binary measure of households moving above or below the poverty threshold and will be quantified by the income deficit across a full range of economic groups for a given fiscal or monetary policy or economic event.

The basket of supplies is stored as a computational object, which enables the carbon footprint to be resolved for all supplies (Section 3.1.1). As the production of supplies requires input supplies, the transport of input supplies, the use of land and the use of labour, the carbon footprint can be aggregated in a manner similar to the supply costs price being calculated. Therefore, the environmental impact of fiscal or monetary policy or economic events can be modelled, and past events can be used to validate the Economy Dynamics analytical framework. Validation of the Economy Dynamics framework against household poverty and environmental impact will be discussed in Paper 08 (see Appendix A).

3.1.9 COMPUTATIONAL ARCHITECTURE

The computationally repetitive nature of the primitive equation within a macroeconomic simulation means that the analytical software architecture is independent of the simulation complexity when using modern computational data management and simulation governance frameworks (see for example Moffatt, 2020). This enables the simulation definition (including interaction between different types of consumers and producers for different sets of resources for any range of goods and services) to be determined by the economy analyst at runtime and not be limited by the software requirements specification provided to the software developer (see for example Moffatt, 2019). This is important for the development of highly complex simulations with algorithm and data interdependencies, as it enables simulation complexity to be built piecewise from smaller individually validated components of the simulation dataset, using the same software architecture as the final simulation requirement. This changes the primary difficulty in developing an applied mathematical approach to macroeconomic forecasting from being a software architecture problem to one of generating suitable datasets to be representative of household and business financial behaviours. The computational architecture and the analytical process will be discussed in Paper 09 (see Annex A).

3.2 ANALYTICAL PROCESS

The computational process of an Economy Dynamics framework simulation is summarised in Figure 10. The process starts by using the primitive equations (Paper 02) to determine income and expenditure of all consumer entities within the simulation. The relationship between the income and the expenditure then determines the financial behaviour of the consumers, which will be predicted by using surrogate models trained against population data for households (Paper 01) and for businesses (Paper 05). Financial behaviour models for the public purse and charity will need to be defined to reflect changes to fiscal policy (Paper 05) and to reflect changes to charitable fund raising and spending (Paper 05). The financial behaviour of the consumer must be resolved for all the subgroups of the four consumer types in all their respective economy elements. Therefore, the financial behaviour is resolved for all household subgroups $i_{\mathrm{e}+}$ in all economy elements i_α; for all business subgroups $i_{\mathrm{B}+}$ in all economy elements i_β; for all public purse government departments i_{G} in all economy elements i_τ; and for all charitable businesses i_{C} in all elements i_ϕ.

The financial behaviour of all consumers (who are also all suppliers) is treated as a decision that needs to be tested as to whether it can be realised. For example, we can choose to sell our house, but it could remain on the market for a long time before a sale is agreed. The financial decisions, which are stored in the computational object i_b, can be to borrow money, buy or sell financial assets, buy or sell land and property, buy or sell intellectual property, buy and sell supplies, and buy and sell a stakeholding in a private business. One financial decision not within i_b is whether to seek employment or advertise for employees. The decision of creating vacancies (or making redundancies) is captured within the business financial decisions and the decision to seek work (or resign) is captured within the household financial decisions. The occupational index i_o of the vacancies will be contained within the business subgroup object $i_{\mathrm{B}+}$ and of the employment skillset will be contained within the household subgroup object $i_{\mathrm{e}+}$.

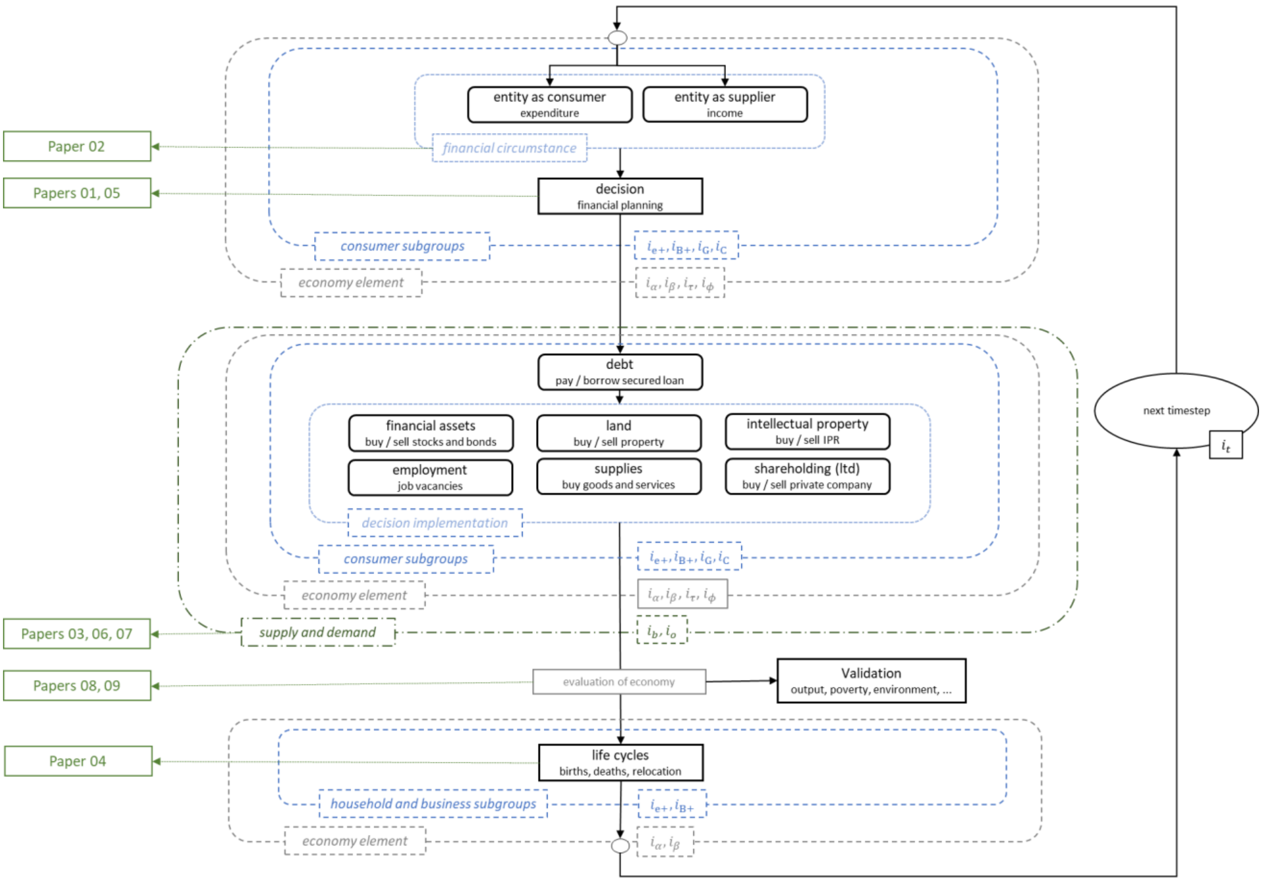

Figure 10. Macroeconomic simulation process

The price of supplies and assets will be resolved based on the supply and demand relationship for supplies in i_b (Paper 03) and for assets in i_b (Paper 06) incorporating other relevant factors. The pay rate for labour and level of employment will be resolved for vacancies in the occupations i_o (Paper

07) using the supply and demand relationship and incorporating other relevant factors. The process for resolving the unit price and volume of sales of supplies and the pay rate and employment level for occupations is achieved by aggregating the decisions of all consumer (and supplier) subgroups ($i_{\mathrm{e}+}, i_{\mathrm{B}+}, i_{\mathrm{G}}, i_{\mathrm{C}}$) and resolving the economy elements of demand onto the economy elements of the supplies (i_β), assets (i_α, i_β, i_τ, i_ϕ) and labour (i_α). In resolving the price of supplies the Economy Dynamics model knows the cost price of generating the supply, it knows the financial circumstances of the business producing the supply, it knows the demand for the supply and the financial circumstances of prospective consumers, therefore it can calculate a realistic sales price. Similar principles can be applied to the sales of assets, including land and property, and the price of labour.

Having resolved the unit and rate prices of supplies, assets and labour and having established the volumes traded the solution for an economy at a given timestep has been generated (Paper 09). The results can be interrogated for validation (Paper 08) against regional and national economic output, poverty and financial welfare levels within the economy elements, and the impact of consumer choice on the environment. In addition, by running the model over a series of timesteps, time derivative values such as the rate of currency inflation, will be generated by the model and can be used for validation. Once the framework has been validated for inflation, the impact of currency inflation can be studied at a regional level (i.e. for any economy element). This would enable fiscal policy to be evaluated that could rebalance the economic benefits of a single currency between regions that benefit and those that are disadvantaged.

Between each timestep the microeconomic structure of the system changes due to the life cycles of households and businesses (Paper 04). The life cycle events have a significant impact on consumer behaviour from the costs of bringing up a child, the demand on education facilities, the preferred choice of housing, people joining the workforce, people leaving the workforce and the redistribution of assets (if any) following a death. These events are resolved as a proportion of the population of each household subgroup $i_{\mathrm{e}+}$ in each economy element i_α and of each business subgroup $i_{\mathrm{B}+}$ in each economy element i_β. The size of the population in each consumer subgroup ($i_{\mathrm{e}+}, i_{\mathrm{B}+}$) in each economy element (i_α, i_β) is adjusted and the aggregated assets/debt value of the i_b items and wage value for employment occupations i_o of each subgroup ($i_{\mathrm{e}+}, i_{\mathrm{B}+}$) in each economy element (i_α, i_β) is calculated. The economy primitive equations are then resolved (Paper 02) starting a new timestep.

4 CONCLUSION

Economy Dynamics avoids the limitations imposed by extrapolating semi-empirical functions. It uses applied mathematics to establish the financial circumstances of consumers and suppliers, which is dependent on fiscal and monetary policy and global events impacting the price of supply inputs into an economy. The microeconomic models may introduce assumptions about consumer behaviour, but since the models are not extrapolated the simulations can be validated against historical data. If the validation fails then the microeconomic models can be replaced or refined. A key microeconomic assumption of Economy Dynamics is that consumer and supplier behaviour is driven by the financial circumstances of the individual entity and not knowledge of the economy. The growth of the economy may be reflected by different proportions of the consumer entities that encounter negative or positive financial pressures, but at any time in any economy there will be entities that experience a wide range of different financial pressures. Therefore, by establishing the entity's financial circumstances using applied mathematics the consumer and supplier behaviour can be modelled without extrapolating semi-empirical functions that have been fitted to population data.

The proposed household behaviour framework uses the welfare metric of the minimum income standard within each economy element, which enables the economic forecast to be validated against recent past population poverty data. This also enables the impact on household welfare of changes to fiscal policy to be assessed. The proposed use of a supply computational object, that can aggregate the supply carbon footprint, enables the environmental impact of consumer behaviour to be evaluated. Thus, forecast environmental impact can also be correlated to the environmental impact over the recent past. Once validated, the model will be able to simulate the environmental impact of any economy structure (including fiscal and monetary policy, and events affecting availability of essential supplies). The proposed used of a time marching methodology means that time derivative values are outputs of the solution and are independent of analyst assumptions. This enables the modelling of inflation of a currency at regional level and not just an aggregated forecast at the economy level.

The Economy Dynamics framework will be used to evaluate the effect of fiscal and monetary policy on economic output, household welfare and environment impact. The methodology will enable the study of the resilience of an economy to external events affecting essential supplies, such as regional political instability or resource depletion. The approach enables the validation of existing economic theories and can supplement historical datasets with forecast results to provide a wider range of input variable values for training current semi-empirical methods. Thus, Economy Dynamics has spill over benefits in delivering greater confidence in current methods, which could extend the use of these methods.

5 FURTHER WORK

The Economy Dynamics framework has been constructed from the experience of a business founder. The author has independently completed three of the ten papers. Paper 00 provides an overview of the Economy Dynamics framework. Paper 01 defines an approach to modelling household decision-making from population data. Paper 02 presents the macroeconomic primitive, interlinking the income and expenditure of all consumer-suppliers. The next stage of the work will require around two man-years of effort between two people to produce the remaining papers and demonstrate the feasibility of a software architecture to enable the development of the adaptable Economy Dynamics analytical framework to allow analysts to interchange different micro- and macroeconomic analytical functions. As the methodology has a low technology readiness level, securing private investment is unlikely. However, there are no public funding routes to develop macroeconomic theories for people who are not employed at a university or at a non-profit research institute. Axsym is currently seeking financial support to continue this project.

Following the proof of feasibility, the next stage of the project will require a proof of concept. Owing to the innovative planned architecture for the Economy Dynamics framework, the labour-intensive activity of the proof of concept will be creating the behavioural and life cycle models. It is planned to demonstrate the proof of concept by validating the framework against a major city such as London or Manchester or other major regional (or national) commercial centre. It is anticipated that this will take around 50 man-years of effort, over a five-year duration. This will require collaboration with existing organisations that are experienced at collating population economic statistical data and could support the development, or could develop, the behavioural and life cycle surrogate models and define the supplies object and the occupational index.

6 REFERENCES

Blanchard, O. J., 2008. The state of Macro. National Bureau of Economic Research Working Paper No. 14259. https://www.nber.org/papers/w14259

Bradshaw, J., Middleton, S., Davis, A., Oldfield, N., Smith, N., Cusworth, L. and J. Williams, 2008. A minimum income standard for Britain: what people think. Joseph Rowntree Foundation. https://www.jrf.org.uk/report/minimum-income-standard-britain-what-people-think

Drzewiecki, M. S., 1892. Method pour la determination des elements mechaniques des propulseurs helicoidaux. Bulletin de l'Association Technique Maritime, pages 10 – 31. https://archive.org/details/bulletindelasso06marigoog/page/n32

Hill, K., Davis, A., Hirsch, D., and L. Marshall, 2016. Falling short: the experiences of families living below the Minimum Income Standard. Inspiring Social Change, Joseph Rountree Foundation. https://www.jrf.org.uk/report/falling-short-experiences-families-below-minimum-income-standard

Moffatt, S., 2019. Handling data in automated analysis processes. Axsym Engineering Publications, Teclab Paper 01: http://www.axsym-engineering.com/publications/Teclab01.pdf

Moffatt, S., 2020. Automating analysis processes for complex modelling ontologies. Axsym Engineering Publications, Teclab Paper 02: http://www.axsym-engineering.com/publications/Teclab02.pdf

OBR.uk, 2013. The macroeconomic model. Office for Budget Responsibility, Briefing paper No. 5: https://obr.uk/docs/dlm_uploads/Final_Model_Documentation.pdf

ONS.gov.uk, 2019. Household final consumption expenditure: National concept CVM SA - £m. Office for National Statistics (UK): Economic, social and population statistics publication: https://www.ons.gov.uk/economy/nationalaccounts/satelliteaccounts/timeseries/abjr/ukea

Turner, M. J., Clough, R. W., Martin, H. C. and L. J. Topp, 1956. Stiffness and deflection analysis of complex structures. Journal of the Aeronautical Sciences, Vol 23 (9).

7 APPENDIX A – ECONOMY DYNAMICS REPORT SERIES

The papers listed below, once ready for public release can be downloaded from: www.axsym-engineering.com/publications.php

Paper 00. Towards a comprehensive macroeconomic simulation capability: Economy Dynamics

An overview of the Economy Dynamics methodology for macroeconomic analysis and forecasting.

http://www.axsym-engineering.com/publications/EconomyDynamics_Paper00.pdf

Paper 01. Modelling decision-making from population data: Household Income Forcing

Household financial decision-making can be captured from population data by using surrogate modelling methods. The probability of a given financial action can then be modelled under different household financial circumstances by replaying the surrogate model using the revised household financial circumstances.

http://www.axsym-engineering.com/publications/EconomyDynamics_Paper01.pdf

Paper 02. The economy in one equation: The Macroeconomic Primitive

All financial interactions can be represented by a single equation defining the interdependency between income and expenditure of all consumers.

http://www.axsym-engineering.com/publications/EconomyDynamics_Paper02.pdf

Paper 03. The price of supplies: Resolving net supply and demand characteristics

Resolving consumer demand and supplier supply across the respective economy regions to determine the supply unit and rate price and the volume of supplies sold.

Paper 04. Consumer life cycles: Microeconomic dynamic change

An analytical framework to capture the probability of household and business life cycle events occurring, such as birth and death, start-up and bankruptcy. The probability of the events occurring are captured from population data by using surrogate modelling methods.

Paper 05. Modelling decision-making from population data: Business Income Forcing

Business financial decision-making can be captured from population data by using surrogate modelling methods. The probability of a given financial action can then be modelled under different business financial circumstances by replaying the surrogate model using the revised business financial circumstances.

Paper 06. The price of land: Resolving net supply and demand characteristics

Resolving consumer demand and asset availability across the respective economy regions to determine the asset unit price and the volume of assets sold.

Paper 07. The wage rate for labour: Resolving net supply and demand characteristics

Resolving employment demand and skills availability across the respective economy regions to determine the wage rate and employment levels against occupation.

Paper 08. Validation of the Economy Dynamics analytical framework

Modelling household poverty and environmental impact of supplies and the processes of comparing the output to observed characteristics of an economy.

Paper 09. Element time marching computational framework for macroeconomic simulation

Defining the computational framework and the simulation process for an applied mathematics approach to macroeconomic forecasting. Also, discusses the software architectural specification to enable interchangeability of theories and functions for forecast validation against observed characteristics of an economy.

8 DOCUMENT INFORMATION

Author: William James Maybury, *BSc PhD CEng*
Title: Towards a comprehensive macroeconomic simulation capability: Economy Dynamics.
Report: Axsym Engineering Publications
Series: Economy Dynamics
Paper: 00
Issue: 1
Date: 22-01-2020

DOCUMENT REFERENCE

Maybury, W. J., 2020. Towards a comprehensive macroeconomic simulation capability: Economy Dynamics. Axsym Engineering Publications, Economy Dynamics Paper 00:
http://www.axsym-engineering.com/publications/EconomyDynamics_Paper00.pdf

CORRESPONDENCE

email: william.maybury@axsym.co.uk

KEYWORDS

Macroeconomic, Simulation, Analysis, Validation, Economy Dynamics

FUNDING

This research did not receive any specific grant funding from agencies in the public, commercial, or not-for-profit sectors.

ACKNOWLEDGEMENTS

This research exploits IP developed in previous projects co-funded by Axsym Limited and Innovate UK (project no. 132456 & 710722) that proved the concept of an innovative computational framework for automating data intensive simulation processes and (project no. 70021) that evaluated the concept of using surrogate modelling methods for the low cost and rapid turnaround of production design work for engineering manufacturers and for reducing costs in industrial processes.

The author is grateful to Dr Stuart Moffatt for providing suggestions to improve clarity in the presentation of the methodology and to Nicola Maybury for providing advice on improving readability for a general audience.